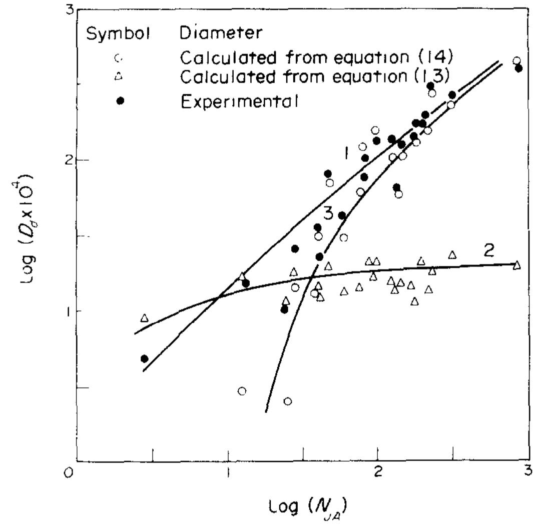

FIG. 3. Experimental and theoretical departure diameters.

It seems, therefore, that for departure diameters less than about 1·6 mm or for Jakob numbers less than about 16, the bubble departure is controlled by surface tension force while for $D_d > 10$ mm (approx.) or $N_{Ja} > 100$ (approx.) the inertia forces control bubble departure. For departure diameters between 1·6 and 10 mm (approx.) or N_{Ja} between 16 and 100 (approx.) the surface tension and inertia forces are of nearly equal importance.

Figure 3 shows D_d as function of N_{Ja}. Curve 1 is a plot of experimental departure diameters while curves 2 and 3 show respectively the values of D_d obtained from the following equations:

$$F_B = F_{ST} \tag{13}$$

$$F_B = F_{CD} + F_{LI}. \tag{14}$$

It is evident that values of D_d predicted by (14) are in reasonable agreement with experiment for $N_{Ja} > 100$ while for $N_{Ja} < 16$ the experimental values are closer to those predicted by equation (13).

CONCLUSION

Equations (13) and (14) predict satisfactorily the bubble departure diameter for $N_{Ja} < 16$ (approx.) or $N_{Ja} > 100$ (approx.), respectively. For intermediate values of N_{Ja} the departure diameter should be computed from equation (10).

REFERENCES

1. C. Y. Han and P. Griffith, The mechanism of heat transfer in nucleate pool boiling—I, *Int. J. Heat Mass Transfer* **8**, 887–914 (1965).
2. A. P. Hatton, D. D. James and T. L. Liew, Measurement of bubble characteristics for pool boiling from single cylindrical cavities, in *Heat Transfer 1970*. Elsevier, New York (1970).
3. R. Cole and H. L. Shulman, Bubble growth rates at high Jakob numbers. *Int. J. Heat Mass Transfer* **9**, 1377–1390 (1966).
4. M. Akiyama, F. Tachibana and N. Ogawa, Effect of pressure on bubble growth in pool boiling, *Bull. J. S. M. E.* **12**, 1121–1128 (1969).

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THERMAL CONVECTION IN A TILTED POROUS LAYER

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NOMENCLATURE

h, thickness of model;
ΔT, temperature difference between lower and upper plane;
g, acceleration of gravity;
x, y, z, dimensionless Cartesian coordinates;
k, m, dimensionless wave numbers in the x- and z-direction, respectively;
Ra, Rayleigh number.

Greek symbols
α, dimensionless overall wave number;
θ, dimensionless perturbation temperature;
φ, tilt angle with respect to the horizontal.

INTRODUCTION

THIS note is concerned with free, thermal convection in a porous layer being tilted an angle φ with respect to the horizontal. The layer is of infinite extent, and is bounded by two impermeable perfectly conducting planes separated by a distance h. The upper and lower planes are maintained at constant temperatures $-\Delta T/2$ and $\Delta T/2$, respectively. Both from a geophysical and technical point of view this type of flow is of considerable interest, and especially the horizontal layer problem is well described in the literature. Concerning a tilted porous layer, however, published works are not numerous. Most recently Bories and Combarnous [1] have studied this problem. Their main experimental results may be stated as follows. At small Rayleigh numbers

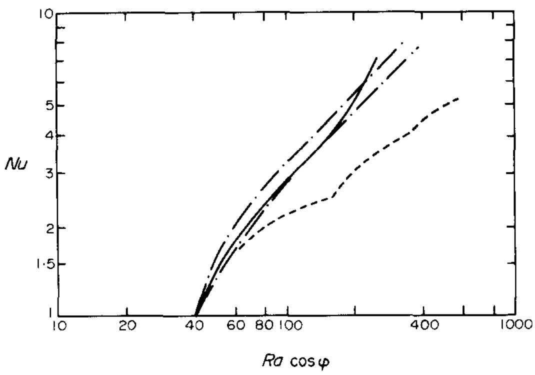

FIG. 1. Values of Nu vs $Ra \cos \varphi$ for longitudinal rolls; ———, the result obtained from Palm, Weber and Kvernvold [7]; —·—·—, limiting curves for the experimental data by Bories and Combarnous [1] for various tilt angles (0–60°); – – –, result from the theoretical analysis in [1].

the motion is unicellular, constituting a basic flow. When the Rayleigh number, or the tilt angle, are sufficiently increased, instability occurs as two-dimensional disturbances with axes aligned in the direction of the basic flow (longitudinal rolls).

ANALYTICAL RESULTS

Let the x-axes be situated in the middle of the layer and tilted an angle φ with respect to the horizontal, and the y-axis be normal to the planes. Hence the basic flow is in the x-direction and varies linearly with y. The temperature profile is also linear in y. Adopting the notation and non-dimensionalisation from [2], we may derive the following equation governing infinitesimal temperature perturbations, θ, in a tilted porous layer

$$\{(D^2-\alpha^2)^2-\sigma(D^2-\alpha^2)-\alpha^2 R + ikR\tan\varphi[y(D^2-\alpha^2)+D]\}\theta(y) = 0 \quad (1)$$

subject to $\theta = D^2\theta = 0$ for $y = \pm\frac{1}{2}$, where $D = \mathrm{d}/\mathrm{d}y$, $Rl = Ra\cos\varphi$ and $\alpha^2 = k^2+m^2$. Here σ is the complex growth rate, Ra the Rayleigh number and k, m real wave numbers in the x- and z-directions, respectively. When φ is small, this equation can be solved by expanding θ in series in $\tan\varphi$. Formally, however, (1) is nearly similar to equation (3.10) in [2] except for a different sign in the last term and one additional term proportional to the square of the small parameter.

Accordingly the results for the present stability problem can be derived from the analysis [2]. We then obtain (i) the principle of exchange of stabilities is valid when φ is small, (ii) the critical Rayleigh number can be written

$$R = Ra\cos\varphi = 4\pi^2+3k_0^2\tan^2\varphi+O(\tan^4\varphi) \quad (2)$$

where $k_0^2+m_0^2 = \pi^2$. Here the subscript 0 refers to the first term in a series expansion in $\tan\varphi$. For a purely two-dimensional disturbance with axis normal to the basic flow (a transverse roll) $m_0 = 0$, while for a longitudinal roll $k_0 = 0$. Accordingly longitudinal rolls minimize the Rayleigh number, and will therefore constitute the preferred mode of disturbance. The occurrence of longitudinal rolls with wave number π, and a critical Rayleigh number given by $Ra\cos\varphi = 4\pi^2$ have been confirmed experimentally in [1]. They also showed that longitudinal rolls constituted a possible stationary solution of the linearized problem.

Actually hexagons were observed in [1] for tilt angles less than about 15°, while longitudinal rolls took over for $\varphi \gtrsim 15°$. The preference of hexagons in a nearly horizontal layer, however, may be attributed to non-linear effects such as the variation of the viscosity and thermal diffusivity with temperature (Palm [3], Busse [4]) or a changing mean temperature (Krishnamurti [5]).

For a nearly vertical layer, observations show that the basic solution is stable, thus supporting the proof by Gill [6].

As shown in [1], the longitudinal roll solution exhibits no x-dependence for moderate values of the Rayleigh number and the tilt angle. Accordingly the non-linear system of equations governing stationary convection in the present problem is identical to that governing two-dimensional convection in a horizontal porous layer, except that the acceleration of gravity is diminished by the factor $\cos\varphi$. Hence the analysis of Palm, Weber and Kvernvold [7] can be applied directly to this problem, substituting $g\cos\varphi$ for g. Only the result for the Nusselt number will be given here, and this is plotted in Fig. 1. For comparison is plotted the experimental and theoretical results obtained by Bories and Combarnous [1].

REFERENCES

1. S. A. Bories and M. A. Combarnous, Natural convection in a sloping porous layer, *J. Fluid Mech.* **57**, 63 (1973).
2. J. E. Weber, Convection in a porous medium with horizontal and vertical temperature gradients, *Int. J. Heat Mass Transfer* **17**, 241 (1974).
3. E. Palm, On the tendency towards hexagonal cells in steady convection, *J. Fluid Mech.* **8**, 183 (1960).
4. F. H. Busse, The stability of finite amplitude convection and its relation to an extremum principle, *J. Fluid Mech.* **30**, 625 (1967).
5. R. Krishnamurti, Finite amplitude convection with changing mean temperature. Part 1. Theory, *J. Fluid Mech.* **33**, 457 (1968).
6. A. E. Gill, A proof that convection in a porous vertical slab is stable, *J. Fluid Mech.* **35**, 545 (1969).
7. E. Palm, J. E. Weber and O. Kvernvold, On steady convection in a porous medium, *J. Fluid Mech.* **54**, 153 (1972).